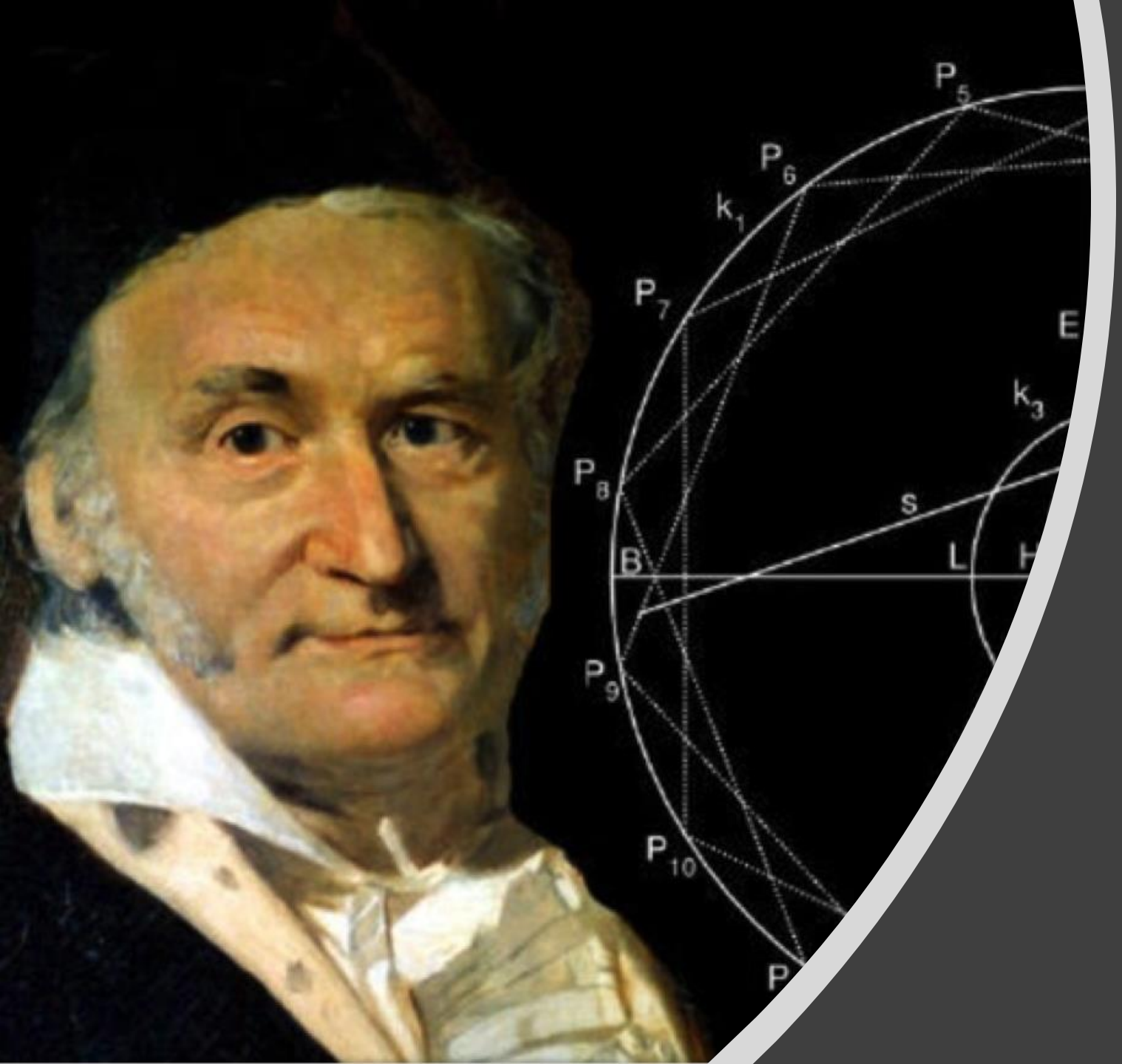

The Remarkable Theorem of Gauss

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Directed Reading Program

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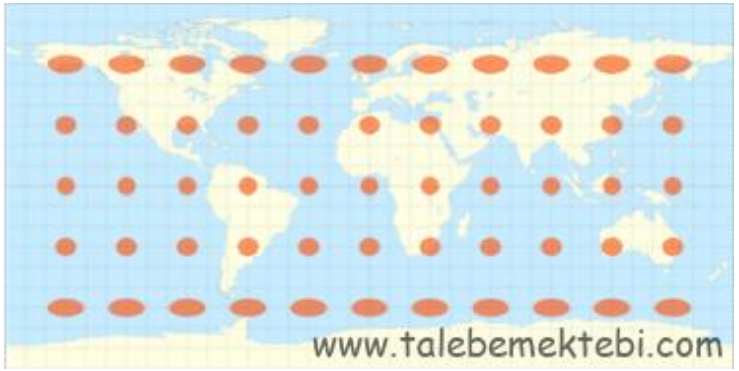
Introduction

- Have you noticed what happens to pizzas when folding it?

- Why all 2D maps of earth are going to be distorted?



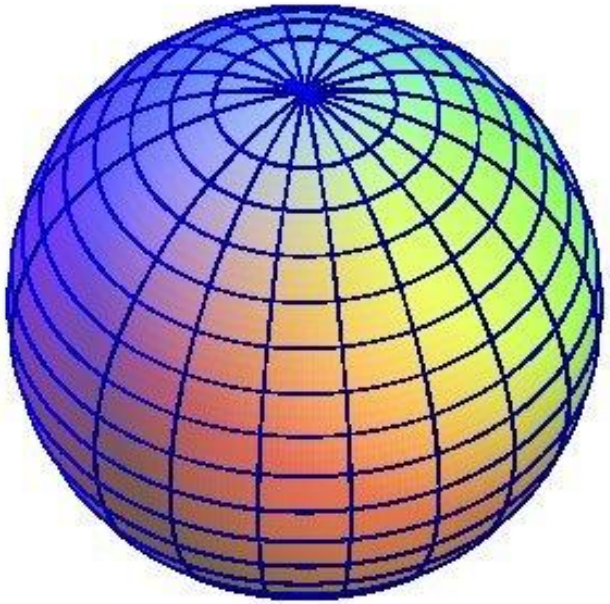
Curvature of A Curve

- A curve is a map from an interval to $\mathbb{R}^3$.
- A regular curve is a curve where its first derivative is never 0.
- The curvature is the acceleration(second derivative) of a unit speed parametrization of a curve. Length is time passed in this case.
- In case not unit-speed, we could use the following derivation:

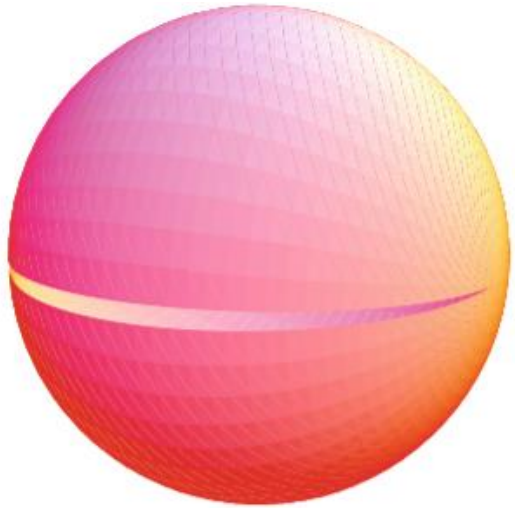
$$\kappa = \frac{\|\dot{\gamma} \times \ddot{\gamma}\|}{\|\dot{\gamma}\|^3}$$

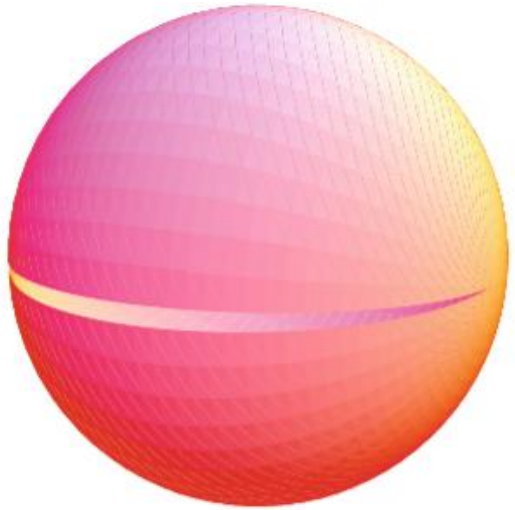
A surface

- A surface is roughly an object embedded in $\mathbb{R}^3$ in such a way that an ant on top of it would think it is a plane.

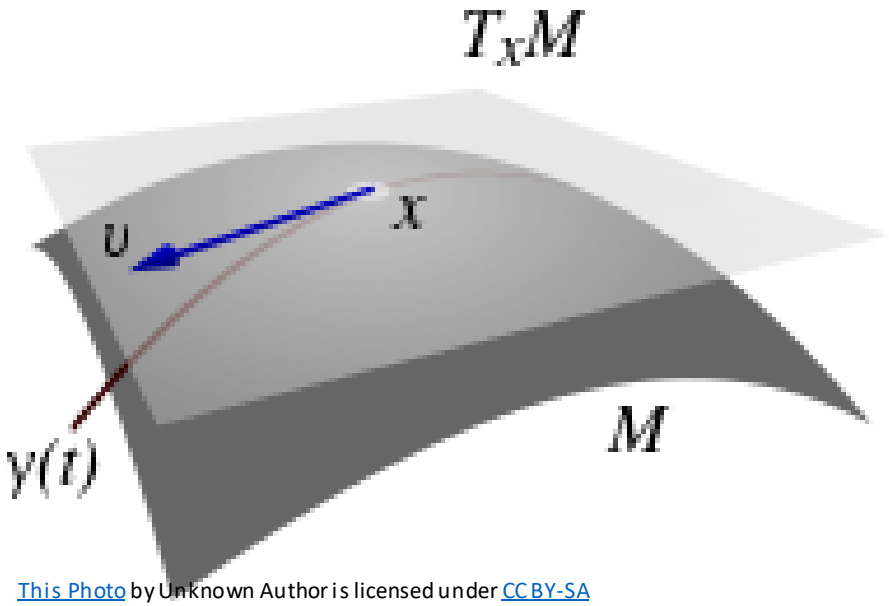


Is sphere a surface?





Tangent space and normal



Local Isometry

- It preserves lengths of curves on a surface.
- Examples are folding a paper or a tissue.

Principal curvature and Gaussian curvature

- Gaussian Curvature is the multiplication of the two principal curvatures
- Principal curvatures are the max and min normal curvatures

Gauss Remarkable Theorem

- Local Isometries reserve the Gaussian curvature!!