The Ising Spin Lattice Model

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Mini Physics Intro Course

Magnets and spins, magnetization

Ferromagnetism

Spins under an external magnetic field

Ising Nearest-Neighbor Lattice Model

Models a ferromagnetic (or antiferromagnetic) material

Only nearest-neighbor interactions

Square Lattice with periodic boundary conditions

1D Ising Model

We want to predict the magnetization of the system $M(H,T) = \langle \sigma \rangle$

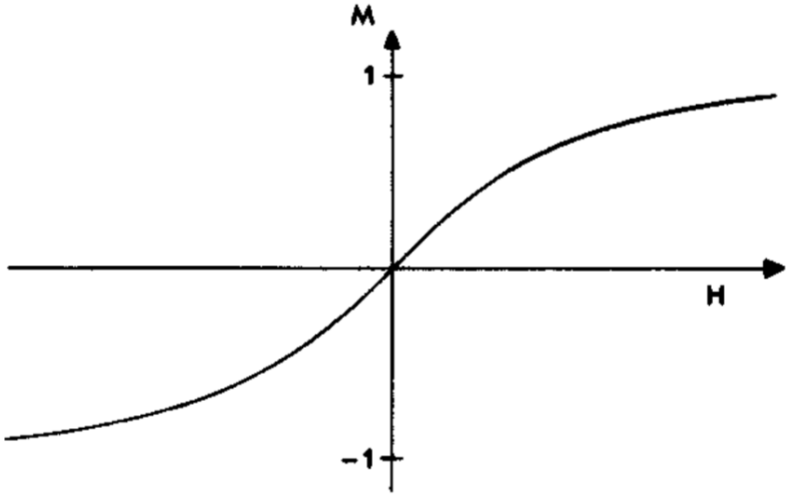
$$Z = \sum_{\sigma} e^{-\beta E(\sigma)} \qquad \text{where} \quad E(\sigma) = -J \sum_{i=1}^{N} \sigma_i \sigma_{i+1} - H \sum_{i=1}^{N} \sigma_i$$

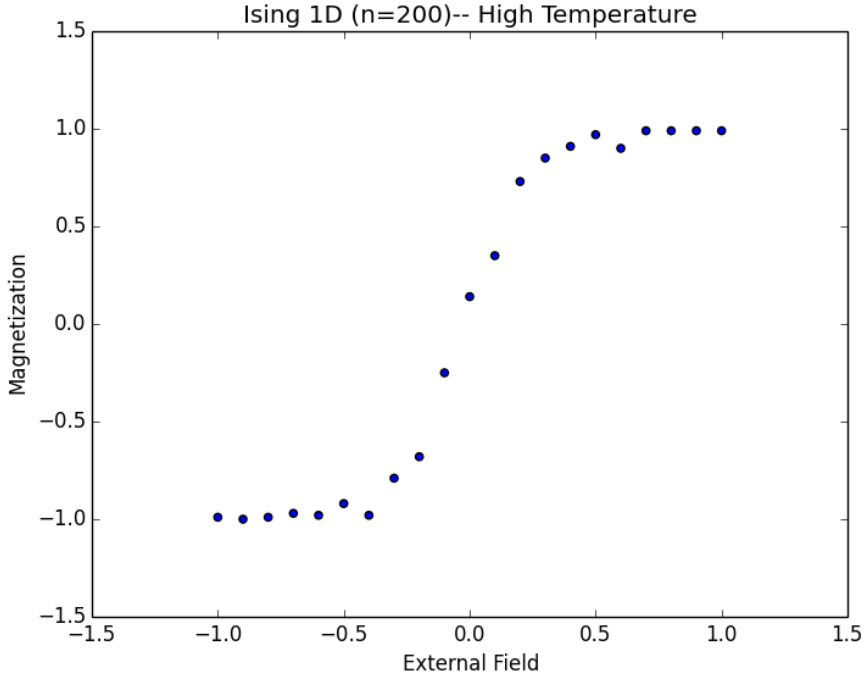
$$f(H,T) = -kT \lim_{N \to \infty} N^{-1} \ln Z$$

$$M(H,T) = \frac{\partial f}{\partial (\beta H)}$$

1D Ising Model

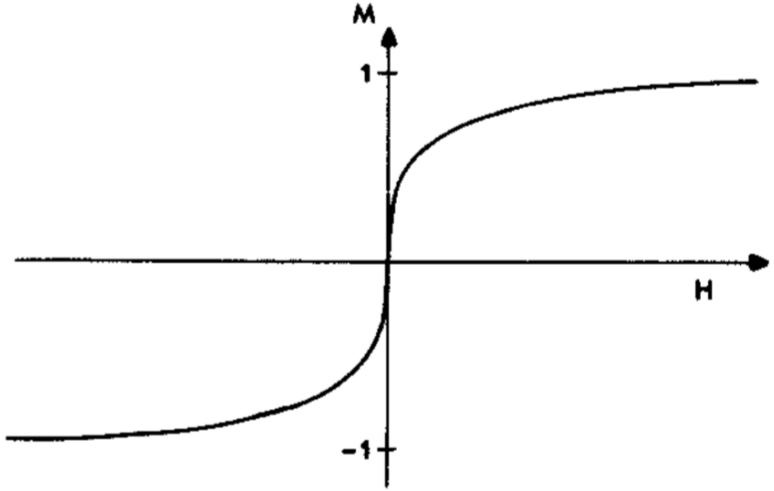
$$M(H,T) = \frac{e^{\beta J}\sinh\beta H}{(e^{2\beta J}\sinh^2\beta H + e^{-2\beta J})^{1/2}}$$

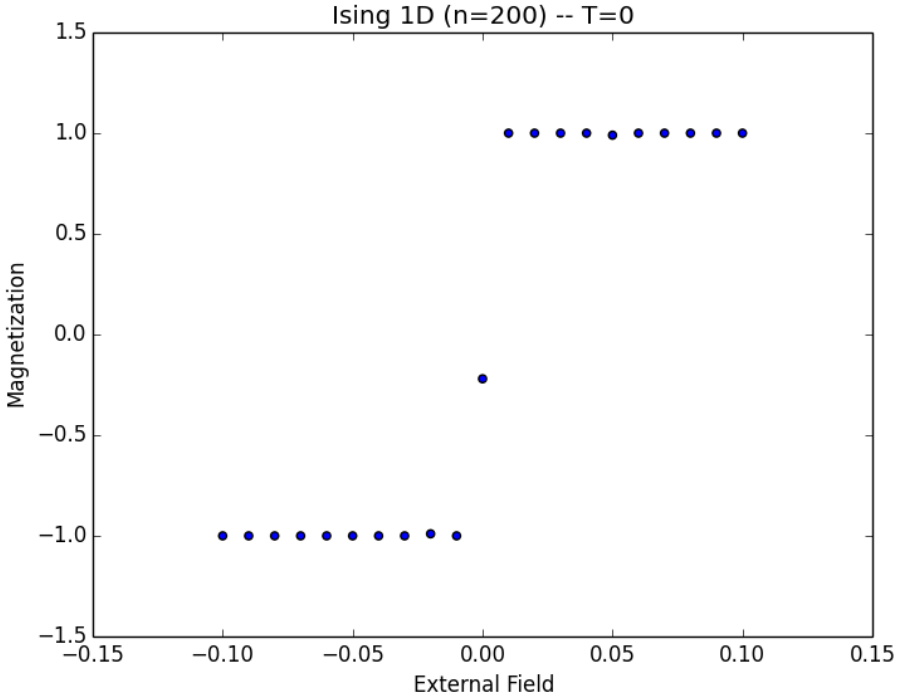




1D Ising Model

Critical behavior at T=0

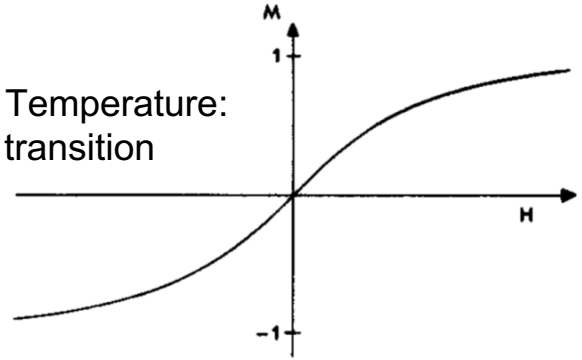


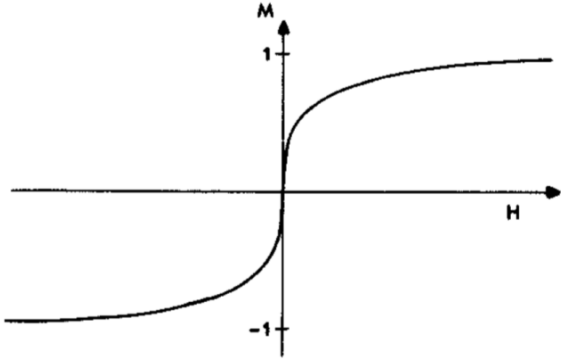


2D Ising Model

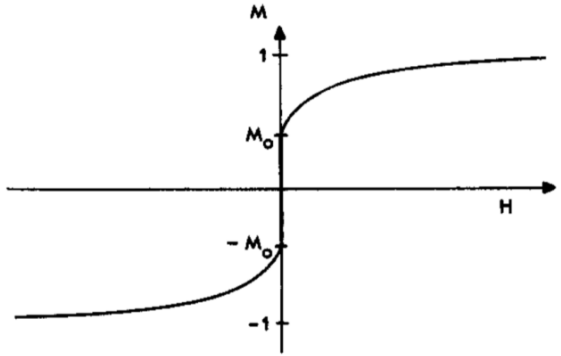
Phase transition for temperatures below the Curie Temperature (Tc > 0)

T > Curie Temperature:
no phase transition



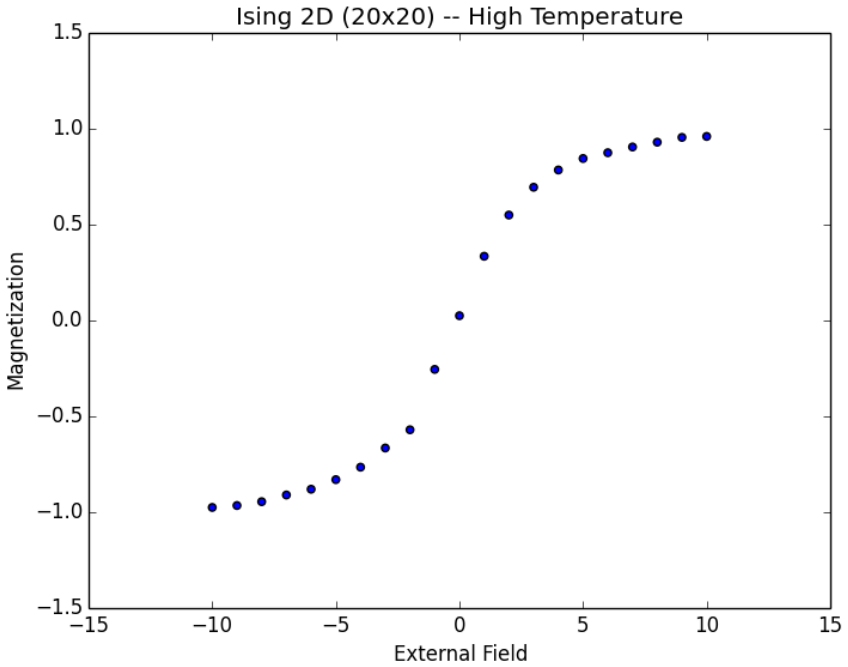


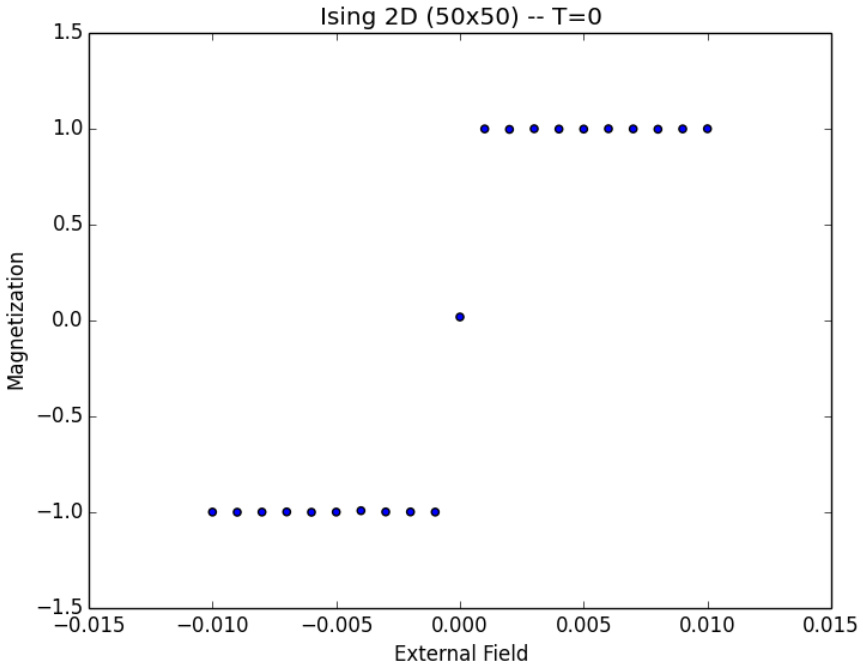
T = Curie Temperature



T < Curie Temperature:
phase transition

2D Ising Model





3D and Beyond

No analytical solution for dimensions 3 and higher

Numerical solutions

(see computer simulations)

Simulations

Step 1: select a location at random

Step 2:

- Metropolis Algorithm:
  - if Energy decreases → flip spin
  - if Energy increases → flip spin with probability $e^{-\beta \Delta E}$
- Glauber Algorithm:
  - Flip spin with probability $1/(1+e^{\beta \Delta E})$
- Voter Algorithm:
  - Set spin to be like a randomly selected neighbor

Applications

Magnets

Neurology

Computer science

Solar magnetograms

Texts and Sources

R.J. Baxter : Exactly Solved Models in Statistical Mechanics

R.A. Minlos : Introduction to Mathematical Statistical Physics (Lecture Series)

Linda E. Reichl : A Modern Course in Statistical Physics (4th ed.)

D.A. Levin : Glauber Dynamics for Ising Model I (AMS Short Course, UOregon)

Raissa D’Souza : Simulating Glauber Dynamics for the Ising Model (UC Davis)